“A Survey on the Four Families of Performance Measures”

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Agenda

- Motivation
- Literature
- The Four Families of Performance Measures
- Preliminary Conclusions
- Extensions
Motivation

Since the introduction of the Sharpe ratio (1966), different measures of portfolio performance are proposed in the literature.

The most complete studies so far are: Aftalion and Poncet (2003), Bacon (2008), Cogneau and Hübner (2009a and 2009b).

The aim of this paper is to present a complete survey of portfolio performance measures, according to four main families:

- Relative performance measures;
- Absolute performance measures;
- Density-based performance measures;
- Utility-related performance measures.
Some Surveys about Performance Measurement


The Four Representative Performance Measures


The Four Families of Performance Measures (1)

**General Form of Relative Performance Measures**
(most often expressed in return *per* unit of risk)

\[
PM_p = \mathcal{P}(r_p^*) \times \left[ \mathcal{R}^*(r_p^*) \right]^{-1},
\]

where \(r_p^*\) are the (rescaled) returns, \(\mathcal{P}(\cdot)\) is a function that depends upon the observed performance, and \(\mathcal{R}^*(\cdot)\) is a corrected risk measure of the investor’s portfolio under study, such as \(\mathcal{R}^*(\cdot) = \mathcal{R}(\cdot) \times c(\cdot)\).

**Objective:** comparing the observed (rescaled) performance of the managed portfolio *per* unit of risk.
The Four Families of Performance Measures (1)

- The Reward-to-variability ratio (Sharpe, 1966)

\[ S_p = \left[ E\left(r_p\right) - r_f \right] \times \left(\sigma_p\right)^{-1}, \]

where \( E(\cdot) \) is the expectation operator, \( r_p \) and \( \sigma_p \) are respectively the returns and the total risk of the portfolio \( p \), and \( r_f \) is the risk-free asset.

Interpretation: this measure evaluates the compensation earned by the portfolio manager, as gauged by the expected excess return per unit of portfolio total risk.
## The Four Families of Performance Measures (1)

### The Main Sharpe-based Performance Measures

<table>
<thead>
<tr>
<th>Author</th>
<th>Name</th>
<th>Rescaled Performance</th>
<th>Risk Measure</th>
<th>Correction Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morey and Vinod (2001)</td>
<td>Double Sharpe Ratio</td>
<td>$E(r_p) - r_f$</td>
<td>Standard Deviation</td>
<td>$(\sigma_s)^{-1}$</td>
</tr>
<tr>
<td>Zakamouline and Koekebakker (2009)</td>
<td>Adjusted for Skewness Sharpe Ratio</td>
<td>$E(r_p) - r_f$</td>
<td>Standard Deviation</td>
<td>$\left{1 + b_{3,p} \times [sk(r_p) \times 3^{-1}] \times {[E(r_p) - r_f] \times \sigma_p^{-1}}^{\frac{1}{2}}\right}$</td>
</tr>
<tr>
<td>Israëlsen (2005)</td>
<td>Reward-to-absolute-excess return Ratio</td>
<td>$E(r_p) - r_f$</td>
<td>Standard Deviation</td>
<td>$1^{sign}[E(r_p) - r_f]$</td>
</tr>
<tr>
<td>Sharpe (1994)</td>
<td>Information Ratio</td>
<td>$E(r_p) - r_f$</td>
<td>Tracking Error</td>
<td>-</td>
</tr>
<tr>
<td>Treynor (1965)</td>
<td>Reward-to-volatility Ratio</td>
<td>$E(r_p) - r_f$</td>
<td>Beta</td>
<td>-</td>
</tr>
</tbody>
</table>
## Performance Measures based on Other Risk Measures

<table>
<thead>
<tr>
<th>Author</th>
<th>Name</th>
<th>Rescaled Performance</th>
<th>Corrected Risk Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darolles et al. (2009)</td>
<td>L-performance</td>
<td>TL-moment of order 1</td>
<td>TL-moment of order 2</td>
</tr>
<tr>
<td>Konno and Yamazaki (1991)</td>
<td>Mean Absolute Deviation Ratio</td>
<td>$E(r_p) - r_f$</td>
<td>Mean Absolute Deviation</td>
</tr>
<tr>
<td>Caporin and Lisi (2011)</td>
<td>Reward-to-range Ratio</td>
<td>$E(r_p) - r_f$</td>
<td>Range</td>
</tr>
<tr>
<td>Young (1998)</td>
<td>Minimax Ratio</td>
<td>$E(r_p) - r_f$</td>
<td>Minimax Portfolio</td>
</tr>
<tr>
<td>Dowd (2000)</td>
<td>Reward-to Value-at-Risk Ratio</td>
<td>$E(r_p) - r_f$</td>
<td>Value-at-Risk</td>
</tr>
<tr>
<td>Sortino and Satchell (2001)</td>
<td>Reward-to $k$-Lower Partial Moments Ratio</td>
<td>$E(r_p) - r_f$</td>
<td>Lower Partial Moment of order $k$</td>
</tr>
<tr>
<td>Martin and McCann (1989)</td>
<td>Ulcer Performance Index</td>
<td>$E(r_p) - r_f$</td>
<td>Average Squared Weekly Drawdowns</td>
</tr>
</tbody>
</table>
The Four Families of Performance Measures (1)

The Main Limits of Relative Performance Measures

- Rankings are consistent only if portfolio returns are elliptically distributed and/or the representative agent has a quadratic utility function (except for Zakamouline and Koekebakker, 2009)

- Use of derivative instruments (fat-tailed and skewed return distributions) or/and strategies with (time-varying) leverage effects yield to misleading conclusions (see Kao, 2002; Amin and Kat, 2003a and 2003b; Gregoriou and Gueyie, 2003)
General Form of Absolute Performance Measures (expressed in return)

\[ PM_p = \Pi \left[ \mathcal{P} \left( r^* \right), \mathcal{P}^{th} \left( r^*_p \mid \Omega \right) \right], \]

where \( \Pi(\cdot, \cdot) \) is a transformation function (generally linear), \( r^*_p \) are the (rescaled) returns, \( \mathcal{P}(\cdot) \) is a function that depends upon the observed performance and \( \mathcal{P}^{th}(\cdot) \) is a function that is related to the theoretical performance of the investor’s portfolio under study, conditionally to a set of information denoted \( \Omega \).

**Objective:** comparing the observed (rescaled) performance of the managed portfolio to its theoretical performance, considering a model.
The Four Families of Performance Measures (2)

The *Alpha* (Jensen, 1968)

\[
\alpha_p^J = \left[ E(r_p) - r_f \right] - \left[ E(r_m) - r_f \right] \times \beta_{p,m},
\]

where \( E(\cdot) \) is the expectation operator, \( r_p \) are the returns of the portfolio \( p \), \( r_m \) are the returns of the market portfolio \( m \), \( r_f \) is the risk-free rate, \( \beta_{p,m} \) is the sensitivity of investor’s portfolio returns with respect to market portfolio returns.

**Interpretation**: this measure assesses the extra-performance, realized by the portfolio manager, given its sensitivity to systematic risk.
The Four Families of Performance Measures (2)

The Main Jensen-type Performance Measures

<table>
<thead>
<tr>
<th>Author</th>
<th>Name</th>
<th>Rescaled Performance</th>
<th>Theoretical Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fama (1972)</td>
<td>1. Selectivity Measure 2. Risk Compensation Measure</td>
<td>$E(r_p) - r_B(\beta_{p,m})$</td>
<td>$r_B(\beta_{p,m}) - r_f$</td>
</tr>
<tr>
<td>Black (1972)</td>
<td>Zero-beta CAPM</td>
<td>$E(r_p) - E(r_z)$</td>
<td>$[E(r_m) - E(r_z)] \times \beta_{p,m}$</td>
</tr>
<tr>
<td>Treynor and Mazuy (1966)</td>
<td>Market Timing Model</td>
<td>$E(r_p) - r_f$</td>
<td>$[E(r_m) - r_f] \times \beta_{1,p,m} + [E(r_m) - r_f]^2 \times \beta_{2,p,m}$</td>
</tr>
<tr>
<td>Henriksson and Merton (1981)</td>
<td>Parametric Market Timing Model</td>
<td>$E(r_p) - r_f$</td>
<td>$[E(r_m) - r_f] \times \beta_{1,p,m} + \max[r_f - E(r_m), 0] \times \beta_{2,p,m}$</td>
</tr>
<tr>
<td>Connor and Korajczyk (1986)</td>
<td>Multi-factor CAPM</td>
<td>$E(r_p) - r_f$</td>
<td>$\sum_{k=1}^{K} E(F_k) \times \beta_{p,F_k}$</td>
</tr>
<tr>
<td>Ferson and Schadt (1996)</td>
<td>Multi-factor Conditional CAPM</td>
<td>$E(r_p) - r_f$</td>
<td>$\sum_{k=1}^{K} E(F_k) \times \beta_{p,F_k}(t)$</td>
</tr>
</tbody>
</table>
## The Four Families of Performance Measures (2)

### Other Miscellaneous Absolute Performance Measures

<table>
<thead>
<tr>
<th>Author</th>
<th>Name</th>
<th>Performance Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Henriksson and Merton (1981)</td>
<td>Non-parametric Market Timing Model</td>
<td>$P_1(t) + P_2(t) - 1$</td>
</tr>
<tr>
<td>Moses <em>et al.</em> (1987)</td>
<td>Diversification-adjusted <em>Alpha</em> Measure</td>
<td>$\alpha_p^I \times (\lambda_p)^{-1}$</td>
</tr>
<tr>
<td>Grinblatt and Titman (1989)</td>
<td>Positive Period Weighting Measure</td>
<td>$\sum_{t=1}^{T} w_t \times (r_{p,t} - r_f)$</td>
</tr>
<tr>
<td>Modigliani and Modigliani (1997)</td>
<td>Risk-Adjusted Performance Measure</td>
<td>$E(r_p) \times (1 + \lambda_{p,m}) - r_f \times \lambda_{p,m}$</td>
</tr>
<tr>
<td>Cantaluppi and Hug (2000)</td>
<td>Efficiency Ratio</td>
<td>$[E(r_p) - r_f] \times [E(r_B) - r_f]^{-1}$</td>
</tr>
<tr>
<td>Muralidhar (2001)</td>
<td>Correlation-Adjusted Performance</td>
<td>$[E(r_i) \times w_i] \times [E(r_B) \times w_B] + [r_f \times (1 - w_i - w_B)]$</td>
</tr>
<tr>
<td>Muralidhar (2002)</td>
<td>Skill, History And Risk-Adjusted Measure</td>
<td>$\left{(E(r_i) \times w_i) \times [E(r_B) \times w_B] + [r_f \times (1 - w_i - w_B)]\right} \times C(S)$</td>
</tr>
</tbody>
</table>
The Main Limits of Absolute Performance Measures

Performance computed with these measures are strongly influenced by the reference portfolio (market portfolio or proxy).

Almost all of them assumes stability of the systematic risk sensitivities of the investor’s portfolio over time (except for Ferson and Schadt, 1996).

Disregards skewness and kurtosis of the studied portfolio returns which may alter rankings when using investment strategies based on derivative instruments (except for Hwang and Satchell, 1998).
General Form of Density-based Performance Measures (expressed in scalar terms – no unit)

\[ PM_p = \mathcal{P} \left( r_p^* \right) \times \left[ \mathcal{P}^{-1} \left( r_p^* \right) \right]^{-1}, \]

where \( r_p^* \) are the (rescaled) returns, \( \mathcal{P}(\cdot) \) is a function that depends upon the observed performance and \( \mathcal{P}^{-1}(\cdot) \) is a measure focusing on a specific (left) part of the support of the density of returns.

Objective: comparing the observed (rescaled) performance of the managed portfolio to an expression depending on its losses.
The Omega measure (Keating and Shadwick, 2002)

\[
O_p = \int_{\tau}^{+\infty} \left[ 1 - F(r_p) \right] dr_p \times \left[ \int_{-\infty}^{\tau} F(r_p) dr_p \right]^{-1},
\]

where \( F(\cdot) \) is the cumulative distribution function, \( r_p \) are the returns of the portfolio \( p \) and \( \tau \) is a threshold.

**Interpretation:** it compares the potential gains of the managed portfolio over its potential losses, both defined according to a threshold.
### The Main Density-based Performance Measures

<table>
<thead>
<tr>
<th>Author</th>
<th>Name</th>
<th>Rescaled Performance</th>
<th>Downside Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Farinelli and Tibiletti (2008)</td>
<td>One-sided Risk Measure</td>
<td>$E[\max(r_p - r_f, 0)]$</td>
<td>$E[\max(r_f - r_p, 0)]$</td>
</tr>
<tr>
<td>Sortino et al. (1999)</td>
<td>Upside-potential ratio</td>
<td>$HPM_{k_1}(r - r_p)$</td>
<td>$LPM_{k_2}(r - r_p)$</td>
</tr>
<tr>
<td>Biglova et al. (2004)</td>
<td>Rachev ratio</td>
<td>$E[r_p</td>
<td>r_p &gt; -VaR_{a_1,p}]$</td>
</tr>
<tr>
<td>Biglova et al. (2004)</td>
<td>Generalized Rachev ratio</td>
<td>$E\left[\max(-r_p, 0)^{k_1}</td>
<td>r_p &gt; -VaR_{a_1,p}\right]$</td>
</tr>
<tr>
<td>Kazemi et al. (2004)</td>
<td>Sharpe-Omega ratio</td>
<td>$E(r_p) - \bar{r}$</td>
<td>$E[\max(\bar{r} - r_p, 0)]$</td>
</tr>
</tbody>
</table>
The Four Families of Performance Measures (3)

The Main Limits of Density-based Performance Measures

- Performance measures are highly related to the threshold (risk-free rate, Minimum Acceptable Return or Value-at-Risk)
- The link with the utility function of the studied agent is not straightforward
- Density are subject to model risk
The Four Families of Performance Measures (4)

**General Form of Utility-based Performance Measures** (expressed in return *per* unit of util)

\[ PM_p = \mathcal{G} \circ \left\{ E \left[ \mathcal{V} \left( r_p^* \right) \right] \right\}, \]

where \( r_p^* \) are the (rescaled) returns, \( E(\cdot) \) is the expectation operator, \( \mathcal{G}(\cdot) \) is a value (or utility) function and \( \mathcal{V}(\cdot) \) is a specific function that depends upon the performance of the investor’s portfolio.

**Objective**: incorporating investor's preferences and risk profiles, through value (or utility) functions, when assessing the portfolio performance.
The Morningstar Risk-Adjusted Return (Morningstar Inc., 2002)

\[ MRAR_p = E\left[\left(1 + r_p\right)^{-\lambda}\right]^{-\frac{12}{\lambda}}, \]

where \( E(\cdot) \) is the expectation operator, \( r_p \) are the returns of a portfolio \( p \), \( \lambda \) (with \( \lambda \neq 0 \)) is the risk aversion coefficient of the studied investor.

**Interpretation:** incorporating the behavior of the agent, through a Power Utility Function, for assessing the portfolio performance, given a risk aversion coefficient.
### The Four Families of Performance Measures (4)

#### The Main Utility-based Performance Measures

<table>
<thead>
<tr>
<th>Author</th>
<th>Name</th>
<th>Transformed Performance</th>
<th>Value (or utility) Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stutzer (2000)</td>
<td>Performance Index</td>
<td>( \arg\max_{\theta \in \mathbb{R}_+} {-\log[E(\cdot)]} )</td>
<td>( \exp[\theta \times (r_p - r_B)] )</td>
</tr>
<tr>
<td>Kaplan (2005)</td>
<td>Lambda measure</td>
<td>( \arg\max_{\theta \in \mathbb{R}_+} {E(\cdot)} )</td>
<td>( \theta \times (r_p - r_B) - \delta )</td>
</tr>
<tr>
<td>Gemmill et al. (2006)</td>
<td>Loss-Averse Performance Measures</td>
<td>( P(r_p \geq 0) \times E(U_1) \times [P(r_p &lt; 0) \times E(U_2)]^{-1} )</td>
<td>( U_1 = \max \left[ \left( r_p - r_B \right)^{k_1}, 0 \right] ) ( U_2 = \min \left[ \left( r_p - r_B \right)^{k_2}, 0 \right] )</td>
</tr>
<tr>
<td>Ingersoll et al. (2007)</td>
<td>Manipulation-Proof Performance Measure</td>
<td>( [(1 - \lambda)\Delta t]^{-1} \times \ln[E(\cdot)] )</td>
<td>( \left( 1 + r_p \right) \times \left( 1 + r_f \right)^{-1} \lambda^{-1} )</td>
</tr>
</tbody>
</table>
The Main Limits of Utility-based Performance Measures

- Strongly dependent on the utility function (exponential, power or logarithmic) characterizing the behavior of the studied investor.

- Highly sensitive to the investor’s attitude towards risk through the risk aversion coefficient (which can be time-varying).

- Rankings obtained with these measures are directly related to the benchmark (risk free rate or proxy).
Preliminary Conclusions

- We propose a Survey of performance measures in a uniform and comprehensive framework through four main families clearly identified, namely relative, absolute, density-based and utility-related performance measures.

- We define these main families according to two essential criteria: the unit in which it is expressed, and the way the measure is built.

- We formulate a general expression for each of the four categories of performance measures, in which most of the performance measures fits.
Extensions

- On our on-going research agenda, we have planned to add new performance measures (20 extra or so), in order to complete our actual Survey.

- Next step will be to perform some empirical applications in order to compare the properties of collected measures (rank correlation tests, persistence, lucky versus star funds studies, etc.).

- We would like also to complement our intuitions about a new performance measure (called “Generalized Performance Measure”), which can be seen as a generalization of the four main categories presented in this paper.
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Thank you for your attention...

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