Mean-Modified Value-at-Risk Optimization with Hedge Funds

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Both academic and practitioner research has questioned the use of mean-variance analysis as a central approach to evaluating the benefits of investing in hedge funds. In contrast, other researchers have considered another framework, that is, using a mean-value-at-risk setting in which downside risk is incorporated into the asset allocation model. In short, the optimal portfolio is selected by maximizing the expected return over candidate portfolios so that some shortfall criterion is met. The literature on value-at-risk has grown considerably in recent years. Uryasev and Rockafellar [1999] propose measuring a mean shortfall or conditional VaR, which is the mean of the returns higher than the VaR. Flavin and Wickens [1998] use a GARCH process to model asset returns. Artzner, Delbaen, Eber, and Heath [1997] argue that their proposed coherent measures of risk have certain desirable properties that VaR lacks. Basak and Shapiro [1998] argue that VaR does not consider the magnitude of loss which exceeds the threshold level. They propose an analytical formula to obtain the portfolio's weights of risky assets, assuming that they are lognormally distributed, by minimizing the losses over a threshold. Keating and Shadwick [2002] develop an omega approach. They show that maximizing the ratio between the density higher than the threshold and the density lower than the threshold is the optimal approach. This omega approach accounts for all the moments of the returns distribution.

In this article, we first introduce a framework based on a working paper by Huisman, Koedijk, and Pownall [1999]. Then we use a Cornish-Fisher [1937] expansion in order to compute the value-at-risk for the left tail of the distribution. We did empirical tests, not shown here, which clearly demonstrate that this modified VaR has a minimum and that risk, measured by volatility alone, is underestimated if the portfolio has negative skewness and/or positive excess kurtosis.¹

THE NORMAL VALUE-AT-RISK APPROACH

The risk of a portfolio composed of financial assets can be measured by the value-at-risk (VaR). VaR, as a measure of risk, has some interesting advantages:

- It is recognized by practitioners.
- It measures downside risk, which is interesting to a risk-averse investor such as a pension fund.
- Many academic studies have been done on the subject.
- We can measure risk with just one easily understandable number.
- It can be used for non-normally distributed assets. We will adjust the value-at-risk method by using an empirical VaR and an analytical VaR, which takes the skewness and the kurtosis into account.

1. The normal value-at-risk approach assumes that asset returns are normally distributed. This assumption may not hold in reality, especially for hedge funds which are known for their non-normal distributions.